

Figure 7-2 Representation of a magnet as an assemblage of small dipoles.

Coulomb's Law

We've mentioned attraction and repulsion, but what is the actual force exerted between two magnetic poles? This force is stated in Coulomb's law,

tric poles? This force is stated in Coulomb's law,
$$F = \frac{1}{\mu} \frac{m_1 m_2}{r^2} \tag{7-1}$$
 Magnetic Permeability is
$$-1 \text{ for water/air}$$

$$\mathbf{Figure 7-3} \quad \text{Effect of magnetic permeability on magnetic field strength.}$$

Field Strength

The magnetic field strength H is the force a unit magnetic pole would experience if placed at a point in a magnetic field which is the result of some pole strength m and where r is distance of the point of measurement from m

$$H = \frac{F}{m'} = \frac{m}{r^2} \tag{7-2}$$

Magnetic Moment

Magnetic Moment

If a bar magnet is placed in a uniform magnetic field H (Fig. 7-4), it will experience a pair of equal forces acting parallel to each other but in opposite directions (a couple). The magnitude of the couple is

$$C = 2(ml)H\sin\theta \tag{7-3}$$

where θ specifies the original orientation of the magnet in the field. The motion produced by the couple is dependent on the magnitude of H as well as the value of θ

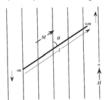


Figure 7-4 Diagram illustrating role of magnetic moment in determining forces acting on a bar magnet placed in a magnetic field.

Intensity of Magnetization

A bar magnet possesses a fundamental property per unit volume known as the intensity of magnetization I. The magnitude of I is defined as the magnetic moment M per unit volume or

$$I = \frac{M}{\text{volume}}$$
(7-5)

and, therefore,

$$I = \frac{ml}{\text{volume}} = \frac{m}{\text{area}} \tag{7-6}$$

Where Magnetic Moment, M, = ml

Magnetic Materials

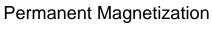


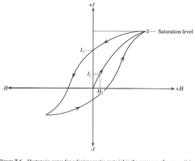
Ferromagnetic

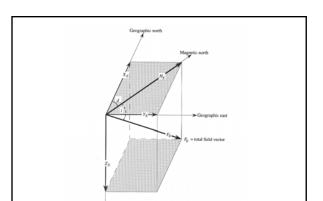
Diamagnetic

Ferrimagnetic/Paramagnetic

Figure 2.5 Schematic representation of magnetic domains in its forestandarie, the set







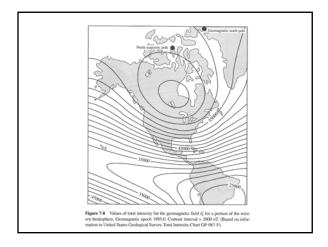
Total, Vertical, & Horizontal Components

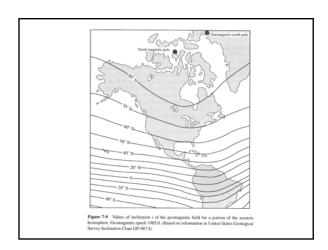
$$F_E = \sqrt{H_E^2 + Z_E^2} = \sqrt{X_E^2 + Y_E^2 + Z_E^2}$$

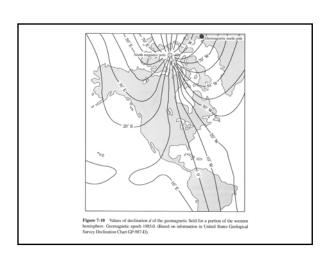
$$Z_E = F_E \sin i, \quad H_E = F_E \cos i, \quad \text{and} \quad \tan i = \frac{Z_E}{H_E}$$

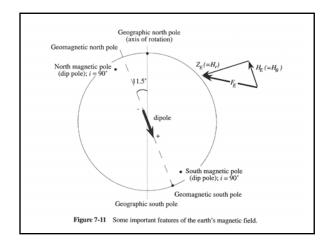
$$X_E = H_E \cos d \quad \text{and} \quad Y_E = \sin d$$
(7-9)

The positions on the earth's surface where $i=90^{\circ}$ are known as the *magnetic dip poles* (see Fig. 7-11), and the *magnetic equator* is defined by positions of $i=0^{\circ}$. At the dip poles $Z_E=F_E$, and the intensity is approximately 70,000 nT (nanotesla). At the magnetic equator, $H_E=F_E$, and the intensity is approximately 30,000 nT. Note that the earth's magnetic field varies in intensity by more than 200 percent, whereas the gravity field varies only by approximately 0.5 percent.









Magnetic Surveys

- Magnetically Clean
- Diurnal Corrections
- Elevation Corrections
- Correcting for Horizontal Position
- Flux-Gate Magnetometer – any component
- Proton-Precession
 Magnetometer total
 only

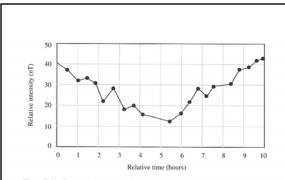
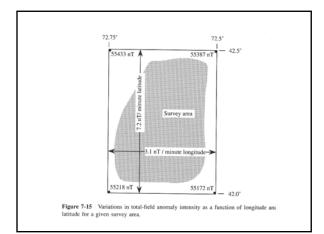


Figure 7-14 Representative variation in magnetic field intensity compiled by returning to a base station at short time intervals during a day's work.



Magnetic Susceptibility

Sedimentary rocks 0.00005 cgs emu
Metamorphic rocks 0.0003 cgs emu
Granites and rhyolites 0.0005 cgs emu
Gabbros and basalts 0.006 cgs emu
Ultrabasic rocks 0.012 cgs emu

Magnetic Potential

Utilizing Eq. 7-2 and remembering that we assume $\mu = 1$, we

have

$$V = -\int_{-r}^{r} \frac{m}{r^2} = \frac{m}{r}$$
 (7-8)

An especially useful feature of the potential is that we can find the magnetic field in a given direction by taking the negative of the derivative of the potential in that direction. We utilize this feature in the following section to derive equations for a dipole.

Dipole Equations

Dipole Equations

The magnetic potential constitutes a convenient approach to describe the magnetic field at a point P due to a dipole. Consider the system illustrated in Figure 7-12(a). We assume that r is much larger than l. From Eq. 7-8 the potential at P is

$$V = \frac{m}{r_1} - \frac{m}{r_2}$$

Using our assumption of r >> l, we have the relations

$$V = \frac{m}{r - \left(\frac{l}{2}\cos\theta\right)} - \frac{m}{r + \left(\frac{l}{2}\cos\theta\right)}$$

$$V = \frac{ml \cos \theta}{r^2 - \left(\frac{l}{2}\right)^2 \cos^2 \theta}$$



$$V = \frac{ml\cos\theta}{r^2} = \frac{M\cos\theta}{r^2}$$
 (7-11)

For our purposes at this time, we wish to derive the radial and tangential components of the field at P (see Fig. 7-12(b)). Recalling that we can determine the magnetic field in a given direction by taking the negative of the derivative of the potential in that direction and noting that θ is in radians, we see that

$$H_r = -\frac{dV}{dr} = \frac{2M\cos\theta}{r^3} \tag{7-12}$$

and

$$H_{\theta} = -\frac{1}{r} \frac{dV}{d\theta} = \frac{M \sin \theta}{r^3} \tag{7-13}$$

$$\frac{dZ_E}{dr} = \frac{dH_r}{dr} = -\frac{6M\cos\theta}{r^4} = -\frac{3}{r}H_r = -\frac{3}{r}Z_E$$
 (7-14)

Total Field Anomaly

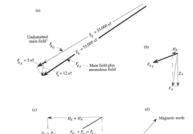


Figure 7-13 Relationships surrounding the meaning of the total-field anomaly. (a) Vectors of the main field and anomalous field. (b) Components of the anomalous field. (c) Components of the undisturbed main field and the anomalous field. (d) Correction for

$$\left(F_E + F_{A_T}\right)^2 = \left(Z_E + Z_A\right)^2 + \left(H_E + H_A\right)^2$$

and

$$F_E^2 + 2F_{A_T}F_E + F_{A_T}^2 = Z_E^2 + 2Z_EZ_A + Z_A^2 + H_E^2 + 2H_EH_A + H_A^2$$
 (7-15)

Because $F_E >>> F_A$, we ignore $F_{A_T}^2$, H_A^2 , and Z_A^2 , which gives

$$F_E^2 + 2F_{A_T}F_E = Z_E^2 + 2Z_EZ_A + H_E^2 + 2H_EH_A$$
 (7-16)

But $F_E^2 = Z_E^2 + H_E^2$, so Eq. 7-16 reduces to $F_{A_T} F_E = Z_E Z_A = H_E H_A$, and

$$F_{A_T} = Z_A \left(\frac{Z_E}{F_E} \right) + H_A \left(\frac{H_E}{F_E} \right)$$
 (7-17)

Finally, we apply the relationships among the geomagnetic elements (Eq. 7-9) to arrive at

$$F_{A_T} = Z_A \sin i + H_A \cos i \tag{7-18}$$

If H_A does not lie along a magnetic meridian, we use the component of H_A parallel to the meridian, because this is the only effect of H_A on the total anomaly. In such a case

$$F_{A_T} = Z_A \sin i + H_A \cos \alpha \cos i \tag{7-1}$$

$$\frac{dZ_E}{d\theta} = \frac{1}{r} \frac{-2M \sin \theta}{r^3} = \frac{-2M \sin \theta}{r^4} = -2\frac{H_E}{r}$$
 (7-20)

Effect of a Monopole

Refer to Figure 7-16 for relevant relationships and notation. These considera-

 $V = \frac{m}{r}$, $m = LA = kF_EA$, and $r = (c^2 + z^2)^{V2} = (x^2 + y^2 + z^2)^{V2}$

Remembering that we determine the magnetic field in a given direction by taking the nega-

$$Z_A = -\frac{dV}{dz} = -\frac{2z(-\frac{1}{2})(kF_EA)}{(z^2+z^2+z^2)^{\frac{1}{2}}} = \frac{z(kF_EA)}{(z^2+z^2+z^2)^{\frac{1}{2}}}$$
 (7-22)

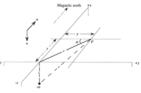


Figure 7-16 Relationships and notation used to derive the magnetic effect of a single note.

Effect of a Monopole

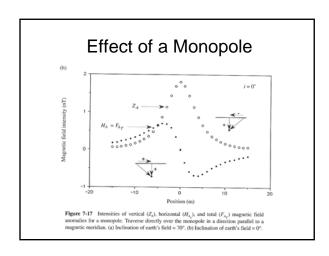
$$H_{A_x} = -\frac{dV}{dx} = \frac{x(kF_EA)}{(x^2 + y^2 + z^2)^{3/2}}$$
 (7-23)

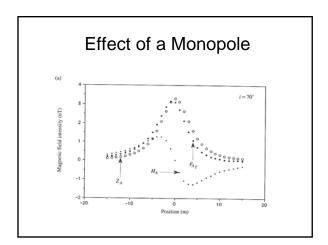
and

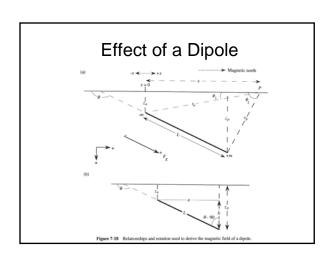
$$H_{A_{\gamma}} = -\frac{dV}{dy} = \frac{y(kF_EA)}{(x^2 + y^2 + z^2)^{3/2}}$$
 (7-24)

The total anomalous field is calculated using the form of Eq. 7-18. Since $H_{\rm A_z}$ represents the component of the horizontal anomalous field in the direction of magnetic north, we have

$$F_{A_T} = Z_A \sin i + H_{A_s} \cos i \qquad (7-25)$$







Effect of a Dipole

Recalling the definition we adopted for a unit pole, the magnetic field intensity at P due to the negative pole of the dipole is

$$R_{A_n} = + \frac{m}{r^2} = + \frac{kF_EA}{r^2}$$
 (7-26)

and that due to the positive pole is

$$R_{A_p} = -\frac{m}{r_p^2} = -\frac{kF_EA}{r_p^2} \tag{7-27}$$

Our next step is to determine the horizontal and vertical components of the magnetic field at P due to each of the poles (-m and +m). These components are

$$Z_{A_{n}} = R_{A_{n}} \sin \phi_{1}, \quad Z_{A_{p}} = R_{A_{p}} \sin \phi_{2}, \quad H_{A_{n}} = R_{A_{n}} \cos \phi_{1}, \quad \text{and} \quad H_{A_{p}} = R_{A_{p}} \cos \phi_{2} \quad (7-28)$$

The final expressions for the vertical and horizontal components simply are the sums of each pole's contribution to the component, or

$$Z_A = Z_{A_a} + Z_{A_p}$$
 and $H_A = H_{A_A} + H_{A_p}$ (7-29)

Effect of a Dipole

The total field is found as in the monopole example except that for the present derivation H_A is oriented parallel to a magnetic meridian, so we can use Eq. 7-18. Now let's place these results in a dynamic table. Once again check Figure 7-18(a) and (b), from which we develop the following relationships:

(1)
$$a = L \cos(180 - \theta)$$
, $b = L \sin(180 - \theta)$, and $z_p = z_n + b$

(2)
$$r_n = (x^2 + z_n^2)^{1/2}$$
, and $r_p = [(x - a)^2 + z_p^2]^{1/2}$

(3)
$$\sin \phi_1 = \frac{z_n}{r_n}$$
, $\cos \phi_1 = \frac{x}{r_n}$, $\sin \phi_2 = \frac{z_p}{r_p}$, and $\cos \phi_2 = \frac{(x-a)}{r_p}$

Effect of a Dipole

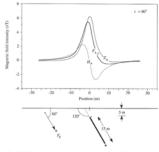
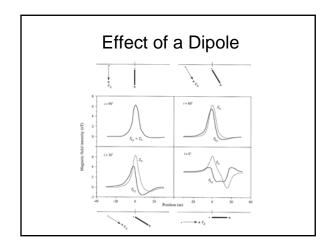
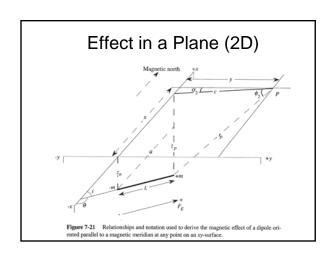
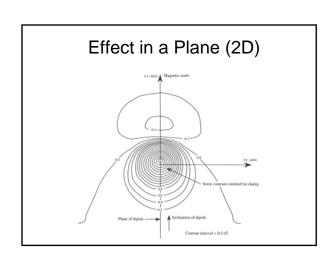


Figure 7-19 Intensities of vertical, horizontal, and total magnetic field anomalies over diseine disole rolarized alone its axis. The disole is oriented parallel to a magneti







Poisson's Relation

The crucial relationship for this derivation is referred to as Poisson's relation (Dobrin, 1988, p. 640–641), which states that the magnetic potential V is proportional to the derivative of the gravity potential U in the direction of magnetization, or

$$V = -\frac{I}{\rho G} \frac{dU}{dw}$$
(7-30)

Remembering that the direction of magnetization for our derivation is vertical or z, it follows that vertical- and horizontal-field anomalies Z_A and H_A must be defined as

$$Z_A = -\frac{dV}{dz} = \frac{I}{\rho G} \frac{d^2 U}{dz^2} \tag{7-31}$$

and

$$H_A = -\frac{dV}{dx} = \frac{I}{\rho G} \frac{d}{dx} \left(\frac{dU}{dz} \right) \tag{7-32}$$

Poisson's Relation

As the gravitational potential of a sphere is

$$U = \frac{GM}{r} = \frac{G\frac{4}{3}\pi R^{3}\rho}{r} = \frac{G\frac{4}{3}\pi R^{3}\rho}{\left(x^{2} + z^{2}\right)^{1/2}}$$
(7-33)

$$\frac{d^2U}{dz^2} = \frac{\left(G\frac{4}{3}\pi R^3 \rho\right) \left(2z^2 - x^2\right)}{\left(x^2 + z^2\right)^{3/2}}$$
(7-34)